

Visualisation of vibration mode shapes to assist students in the learning of mechanical vibrations

Andy C.C. Tan, Gregory D. Paterson, Joseph Mathew & Matthew Dunbabin

Queensland University of Technology
Brisbane, Australia

ABSTRACT: Students often experience difficulties with the subject of mechanical vibrations. One contributing factor has been their inability to observe and understand the physical phenomenon of vibration. The paper describes the design of an innovative laboratory-based system to assist these students. This system illustrates basic vibration theory learnt in class and allows students to perform experimental investigations to physically observe vibration mode shapes generated using external excitation. A theoretical analysis and the application software *ANSYS* were used to develop models to determine the natural frequencies and mode shapes. The results from the modal impact hammer tests and a shaker table were used to provide visual observation and to allow comparisons with the theoretical analysis. This project is aimed at reinforcing students' understanding of vibrations by relating the mathematical theory learnt in class to actual physical systems.

INTRODUCTION

Vibrations are the oscillations of a mechanical system about an equilibrium position and are caused by restoring forces or moments within the system. The energy imparted by an external source lead to vibrations and, if uncontrolled, can result in catastrophic failures of a mechanical system. Rao reported that mechanical failure of machinery is a major cause of large industrial accidents that account for about 38% of total accidents [1]. Since the collapse of the Tacoma Narrow Bridge in the USA in 1940, vibration theory has advanced so much that engineers and scientists are able to control or eliminate its detrimental effects.

Vibration theory is a complex subject and students often experience difficulties with the subject. For a number of years now, instructors in mechanical vibrations have been working to find better ways to assist students to relate theory learnt in class with physical models. To enable students to visualise the phenomena of vibrations, Parker developed an experimental rig that allowed students to relate vibration mode shapes obtained from Finite Element Modelling (FEM) and validation of the results using physical simulations [2]. Daniel and co-workers integrated physical models and computer models into a comprehensive, coherent and understandable practical testing of a two-degree-of-freedom model of an aircraft wing [3]. This model was driven by a motor to simulate the turbine engine of a real aircraft.

At Pennsylvania State University, Perez-Blanco and colleagues developed a series of laboratory sessions to demonstrate the concepts of dynamics and vibrations to undergraduates studying at the university [4]. The practicals allow the students to measure displacement and calculate damping characteristics and natural frequency. They reported that 70% of the students indicated that the laboratory enhanced their understanding of the subject.

COMPUTER SIMULATION

An economical and flexible approach to assist student learning is computer simulation. The Longitudinal Vibration Simulator (LVS) is a potentially useful computer application that allows students to enter system parameters for different beam configurations and watch the resulting animations of stress field distributions. Slater and Gramoll applied the simulator to allow students to explore experimental learning where traditional teaching methods can sometimes appear to be unclear and confusing [5]. By using the program, students can control the cyclic time and obtain animations of the beam motion as well as readings for calculation of motion.

Askari and Davis report on the application of *Visual Basic* to determine the dynamic characteristics of a beam subjected to rotating unbalanced forces [6]. The experimental rig reduces the students' time to perform tedious hand calculations and graphical plots, thus providing valuable time for the students to learn the subject.

Finite Element Analysis (FEA)

A method widely used in the design process of products and machinery is Finite Element Analysis (FEA) [7]. Its main benefits include the prototyping and developing of machinery without incurring heavy capital costs in the manufacturing of real models. Vibration analysis can also be performed with an FEA model. However, the reliability and accuracy of the technique depend on the construction of the model in order to represent the true behaviour of real systems and can, at times, be questionable. Physical tests involving impact hammer method of vibration testing provides a relatively quick, accurate and cost effective method in validating FEA models.

In this paper, the vibration testing of 1DOF and 3DOF structures is used to provide students with visualisation of vibration mode shapes. The models were developed to provide a physical representation of real-life systems analogous to some of the spring-mass systems theory taught in the class. Experimental tests were performed on these models to determine the natural frequencies and vibration mode shapes. The model was attached to a shaker table that was excited using a sine wave generator swept over a range of frequency. When the excitation frequency was close to a natural frequency of the test structure, it was observed that the structure exhibited large deflections.

Mode shapes of the vibrating structure were visualised and captured using stroboscopic methods. The mode shapes were compared with those measured and those determined from theory using FEA. The FEA animations allow students to compare the analytical response to that observed on the shaker rig. At the higher frequencies, the modal deflections of the structure were small, and hence were difficult to observe and capture photographically. However, touch and sound permitted the physical realisation of the modal deflections at these higher resonant frequencies.

EXPERIMENTAL METHOD

The first step of the learning process involved vibration testing using an impact hammer to determine the natural frequencies and mode shapes. Impact hammer theory states that if an object is impulsively struck, it is simultaneously excited at a range of frequencies. Controlling the impact period can change the frequency span of interest. Figure 1 shows the experimental apparatus.

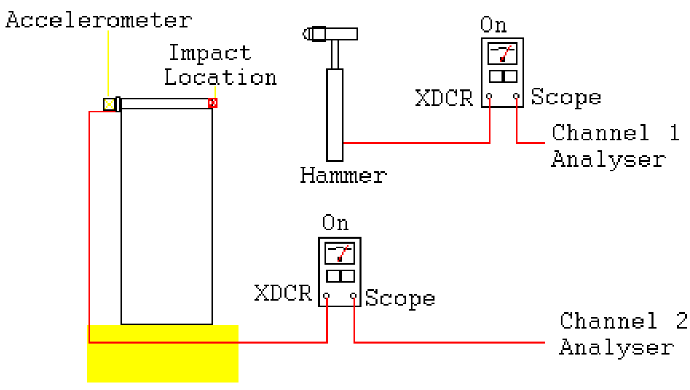

Figure 1: Measurement of a 1DOF structure's natural frequencies.

Measuring the impact force (with a force transducer mounted just behind the hammer tip), and the positional response of the excited object (with an accelerometer), allows a frequency response curve to be obtained for the object. The Dynamic Signal Analyser (DSA) converts the measurement from the time domain to the frequency domain. The frequency response curve can be used to determine the object's behaviour such as its natural frequencies, damping ratio and mode shapes.

The impact hammer used was *modally tuned*. Modal tuning involves a suitable structural design for the hammer that enables the hammer's response to be isolated from the structural response. This enables an accurate measurement of the structural response and not the combined system (impact hammer and structure) response.

MODELLING

In order to overcome the lower frequency limit of the signal analyser, the structure's thickness was reduced to 2mm. For visualisation of mode shapes a 1mm structure was used. Figure 2 shows a wire frame drawing of the 1DOF and 3DOF models. The models were constructed using thin steel strip of 0.364 m in height, width of 0.025 m and thickness of 0.002 m.

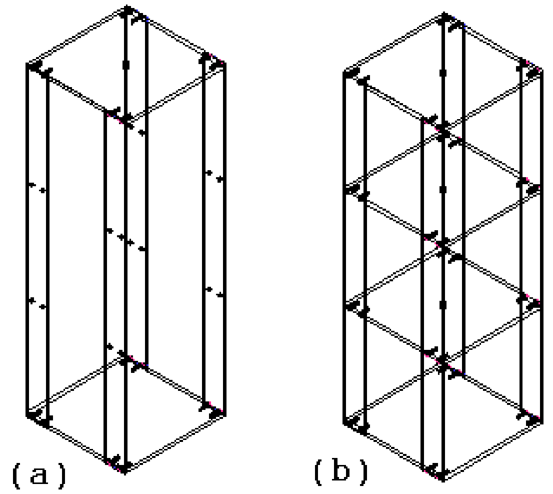

Figure 2: Wire Frame Models of the Structure, (a) 1DOF and (b) 3DOF.

The equations of motion for the system can be written using methods outlined in Steidel [8]. The method used here is to sum the spring forces that exist in the horizontal direction due to a displacement of the mass. The free body diagram is seen in Figure 3. The effective spring constants, $k_1 = k_2 = k_3 = k$ due to the lengths being equal between the masses. The equations of motion according to Newton's Law in matrix format is given by:

$$[M]\{\ddot{X}\}+[K]\{X\}=\{0\} \qquad (1)$$

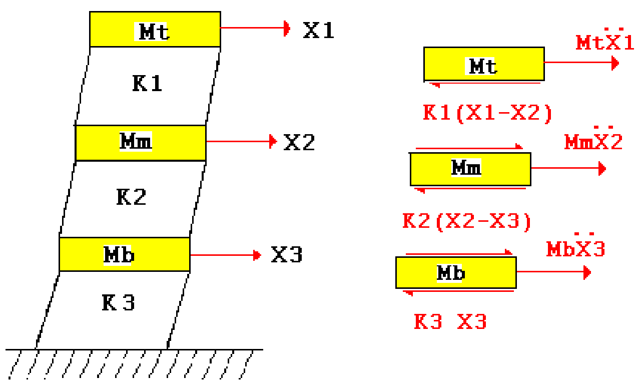

Figure 3: The 3DOF model (left) and the free body diagrams.

With the springs mass taken into consideration the equation with effective mass is given by:

$$[M_{EFF}]\{\ddot{X}\}+[K]\{X\}=\{0\} \quad (2)$$

Providing that the vibration is assumed to be simple harmonic motion, then Eqns (1) and (2) become:

$$([k] - \omega^2[M])\{X\} = 0 \quad (3)$$

and

$$([k] - \omega^2[M_{EFF}])\{X\} = 0 \quad (4)$$

respectively. By solving the matrix equations, the first three natural frequencies are found. The eigen values can be calculated using *MATLAB*.

RESULTS

The experimental tests involved placing additional masses onto the three plates and measurement of the natural frequencies. The results for the 13 tests using the effective mass matrix are shown in Table 1.

Table 1: Modal frequencies.

Vibration Modes	Tests (Hz)	Calculations (Hz)
1st Mode	15-28	15-29
2nd Mode	42-78	42-82
3rd Mode	60-112	61-119

The experimental set-up for the visualisation model is as shown in Figure 4. A shaker with function generator is used to excite the structure in a horizontal manner. Visualisation of the mode shape is obtained using a stroboscope that froze the motion of the model at resonant frequency. If the excitation frequency is at the stroboscope flashing frequency, the oscillating model will be seen to vibrate in slow motion.

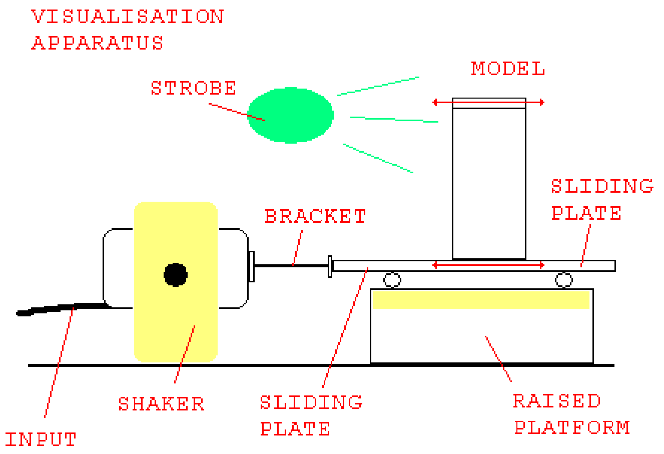

Figure 4: Visualisation experiment set-up.

The mode shape representations measured for the 1DOF and the 3DOF models are shown in Figure 5. Most striking in this rendition of the mode shapes is the slope at the positions of fixture to the plate elements. In nearly all cases, the slope is zero at the fixture elements (positions 3, 6 and 9 correspond to the bottom, middle and top floors). This is a sign that the fixture is not entirely *built-in*, which is the basis for calculation of the spring modulus.

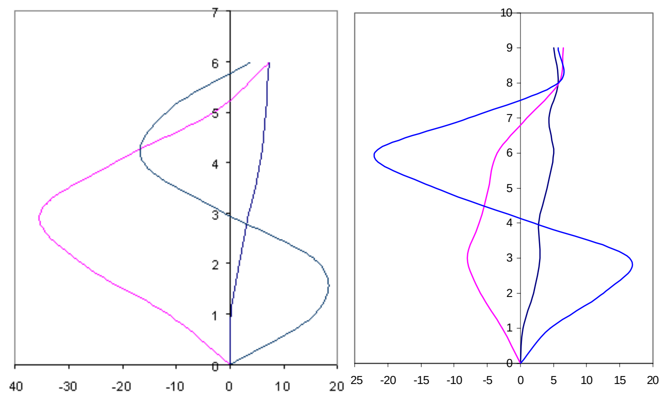

Figure 5: Mode shapes for the 1DOF and 3DOF structures.

DISCUSSION

Adding masses onto the floors of the structure lowers the natural frequencies for both the 1DOF and 3DOF models. A practical relevance of the experimental rig is that the students can relate these tests to real-life structures and by altering the mass, the structure's natural frequencies can be altered to avoid catastrophic failures.

The Finite Element Models gave the natural frequencies as outlined above in Tables 2 and 3. Both the Beam 189 and Solid 187 provided similar animations of the mode shaped and these matched the mode shapes actually seen in the visualisation process. The results obtained using the Beam 189 model were obtained with much less modelling input, and also required less expensive computing time. The Solid 187 model required a very fine mesh and took longer computer time. Symmetry was used to reduce the excessive computing times, which were of the order of three minutes compared to almost instantaneous results obtained using the Beam 189 model. Torsion was seen in the Solid 187 models.

Table 2: Natural frequencies of the 1DOF structure.

Mode	Beam 189	Solid 187	Measured
1	11.634	11.631	10.75
2	88.241	88.241	87
3	232.20	232.20	231

Table 3: Natural Frequencies of the 3DOF Structure.

Mode	Beam 189	Solid 187	Measured
1	28.651	28.543	26
2	81.481	82.441	74.5
3	119.652	124.711	113.5

The effect of the upright mass and the added masses were investigated. When the spring mass was large in proportion to the total mass, large errors in the mathematical models were observed. When the added mass was increased, the measured properties became closer to the predicted values. Furthermore, as the mass was increased, the deflections were smaller and hence reduced the non-linear behaviour in the small mass models.

The first three modes were visible for both structures using the visualisation apparatus. These were achieved using a strobe set flashing as the same frequency of the signal generator. Both

1DOF and 3DOF structures displayed large deflection at the first mode and were clearly visible.

Figure 6 shows a typical mode shape of the 1DOF structure vibrating at the first mode. Higher modes were difficult to capture photographically, but could be either seen or else established by the use of fingers in order to find the positions of zero displacement. All modes were found within a small range of frequencies around those that were found using the impact hammer and those determined from *ANSYS* animations.

Figure 6: Deflection of the 1DOF structure.

CONCLUSION

The modal impact hammer technique was shown to be an economical tool for teaching the vibration behaviour of simple structures to students. The resonant frequencies of both single and multiple degree of freedom models were easily measured using this particular method. This enables the mode shape behaviour to be observed at/or near the predicted natural frequencies. Mathematical models taught in an undergraduate vibration course were applied to the models.

Finite Element Analysis (FEA) provided a further validation of the resonant frequencies and mode shapes. Instructions were produced enabling students to recreate the models using *ANSYS* software. Vibration behaviour exhibited in the laboratory could be observed to be similar to the behaviour predicted by the *ANSYS* software package.

Computer simulation provides the link between mathematical theory and the phenomena observed and measured in the laboratory. This experiment, therefore, addresses the fundamental problem encountered in the learning of vibrations, which is a difficult and highly mathematical subject. Visualisation of structural vibration can provide a better learning experience and generate positive learning outcomes.

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